loci of the roots of $f(z)$, and also where we consider not merely the first but the k^{th} derivative of $f(z)$. The writer hopes to continue the study of these problems.

[1] See paper by the writer, *Trans. Amer. Math. Soc.*, **22**, 1921 (101–116).
[2] See a forthcoming paper by the writer in the *Trans. Amer. Math. Soc.;* a similar result holds for any number of equal circles C_i which have collinear centers.
[3] This problem is set forth explicitly in the paper referred to in the preceding footnote.

ON THE DISTRIBUTION OF THE VELOCITIES OF STARS OF LATE TYPES OF SPECTRUM

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The recent determination of spectroscopic paralaxes or stars of the later types of spectrum made by Adams and his collaborators[1] at the Mt. Wilson Observatory has made it possible to compute the absolute motions in space of about 1300 stars of spectral types A_7 to M for which the proper motions and radial velocities are known. The spectroscopic parallaxes for which the errors are proportional to the parallaxes themselves are in general better adapted for this purpose than the trigonometric parallaxes since the accuracy of the smaller parallaxes is considerably greater.

The three velocity-components of these stars have been computed with reference to the galactic system of coördinates, the pole of galaxy being assumed to have the position A = 190.6°; D = + 27.2°. This system of coördinates is very convenient in the study of the systematic motions of the stars since these motions are mainly in the galactic plane.

Solar Motion.—The solar motion can be obtained directly by forming the algebraic mean of the velocity-components which are measured with reference to the sun. By taking the opposite vector we find the sun's velocity relative to the group of stars employed. The following values for the sun's velocity have been derived in this way.

SPECIAL TYPE	M	NO.	A_0	D_0	V KM.
A6 to F9	<3	153	269°	+27°	21.2
	>3	133	274	35	23.4
G0 to G9	<3	224	266	32	19.9
	>3	154	279	23	40.2
K0 to K9	<4	325	282	43	18.3
	>4	112	286	31	33.1
M	<4	98	271	44	16.2
A6 to M	Giants	800	273	37	18.8
A6 to M	Dwarfs	415	281	29	31.7

The stars of each spectral type have been divided into two groups according to their absolute magnitude M, and these groups, in the case of the later types at least, can be identified with the giant and dwarf classes. We see that for the giant stars the declination D_0 of the sun's apex shows a steady increase from 27° to 44° with advancing spectral type. The same result has been found by several investigators who have studied the distribution of proper motions, and the effect seems to be very marked among the apparently faint stars studied by Dyson and Thackeray.[2] This indicates a larger proportion of stars belonging to the second stream among the K and M stars than among those of early types. The dwarf stars show a much larger value of the velocity of the sun than that deduced from the stars in general.

A very remarkable feature is the behavior of the stars of high velocity. If we group the stars according to their velocities with reference to an origin corrected for the standard solar velocity of 20.0 km. directed toward the point A = 270°; D = + 30°, we find that the velocity of the sun increases remarkably with increasing speed of the stars relative to which its motion is referred.

The following table gives the result of such a computation, the stars being grouped according to velocity.

V KM./SEC.	A_0	D_0	V KM.
0 to 60.........	272°	+30°	20.6
60 to 100.........	295	+43	36.3
100 to 150.........	289	+39	76
>150.........	313	+54	209

We see from this comparison that the stars of high speed have a systematic motion which differs from that of stars of moderate speed, and that they have a tendency to move in a direction opposite to the sun's motion. The same result was indicated by Adams and Joy[3] who found from a study of the space-motions of 37 stars of high radial velocity that nearly all the apices were confined to one hemisphere.

Distribution of Velocities.—The peculiarities in the motions of the stars can best be studied by determining the frequency-function according to which the velocity-vectors are distributed. Two different types of frequency-functions have been used hitherto, namely, those based upon the two-drift theory of Kapteyn[4] and the ellipsoidal theory of Schwarzschild.[5] If use is made of three-dimensional velocity-vectors it is easy to decide between these two theories, but in order to represent the actual distribution of the velocity-vectors in all its aspects a more general type of frequency-function is necessary. In the first place the velocity-vectors were all corrected for a standard solar velocity of 20.0 km. in the direction A_0 = 270° and D_0 = + 30°. About 100 stars which had been selected for observa-

tion on account of their small proper motions were omitted since they would tend to give a "star stream nearly parallel to the direction of the sun's motion. Stars recognized as members of the Taurus Group were also omitted, as well as fainter components of double stars.

The frequency-function used is a three dimensional cosine-series with the origin in the center of the limiting planes. Such a function can be written

$$F(xyz) = a_0 + a_{100}\sin\alpha + a_{010}\sin\beta + a_{001}\sin\gamma + a_{200}\cos 2\alpha + a_{110}\sin\alpha\sin\beta + a_{101}\sin\alpha\sin\gamma + a_{020}\cos 2\beta + a_{011}\sin\beta\sin\gamma + a_{002}\cos 2\gamma + a_{300}\sin 3\alpha + a_{210}\cos 2\alpha\sin\beta + a_{201}\cos 2\alpha\sin\gamma + \ldots.$$

$$\alpha = \frac{\pi x}{2c_1} \qquad \beta = \frac{\pi y}{2c_2} \qquad \gamma = \frac{\pi z}{2c_3}$$

In these expressions $\pm c_1$, $\pm c_2$ and $\pm c_3$, are the six limiting planes within which the function can be regarded as an interpolation formula in three dimensions. For different values of the density F we obtain a sequence of equifrequential surfaces. We see then that α, β, and γ vary between the limits $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$. The integration between certain limits as well as the determination of the coefficients in the series can be performed easily. All terms up to the sixth order have been computed, while in the case of the dwarf stars, where the limits are very large, terms up to the 10th order have been included. The direction of the x-axis is towards the intersection of the galactic plane with the equator, (in Aquila), the y axis is in the direction of galactic longitude 90°, and the z axis towards the galactic north pole.

After the coefficients in the trigonometric series had been computed, a synthesis was made for $z = 0$ and $y = 0$. Curves of equal frequency were then drawn in the xy plane and the xz plane. These curves are the intersections of equifrequential surfaces with these two planes. The total number of stars including those outside the limits have in all cases been reduced to 1000.

The general appearance of the frequency surfaces can be easily seen from the diagrams in figure 1, which give the intersections of the former with the xy and xz planes for the brighter stars of spectral types F to G, and for the dwarf stars of types G0 to M. The curves for the brighter K and M stars are similar to those for the types G_6 to K_1, although the curves are more nearly circular. The crosses (+) represent the velocities of stars belonging to the Taurus Group which were omitted in the analysis. The symbol ⊙ represents the velocity of the sun, whose velocity components in our system of coördinates are $x = +17.0$; $y = +7.4$; $z = +7.4$ km./sec.

The general conclusions to be drawn from a study of these diagrams and the series on which they are based are the following:

1. The giant stars of all types form one single "stream" with an ellip-

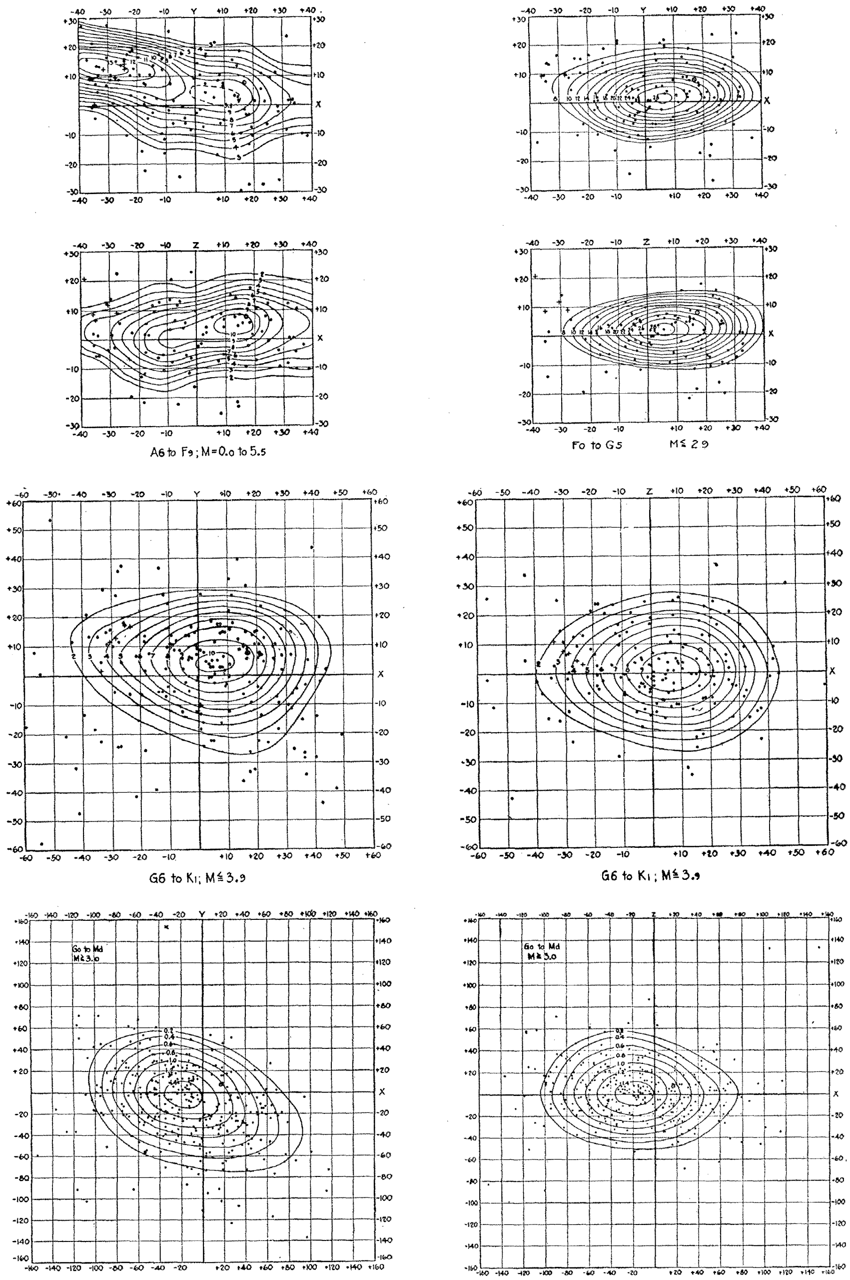

FIG. 1

Distribution of velocities of stars showing the intersections of equi-frequential surfaces of the terminal points of the velocity-vectors with the xy and xz planes.

soidal distribution; the longest and the intermediate axes in all cases lying very nearly in the galactic plane and coinciding approximately with the x and y axes. The shortest axis is perpendicular to the galactic plane. The elongation of the ellipsoids is largest for the bright F stars and decreases for later types, the distribution being nearly spherical for the M stars.

2. The most frequent velocity does not coincide with the origin but lies in all cases in the first quadrant of the xy plane, and has the approximate values $x = +5.6$; $y = +2.3$; $z = +1.6$ km./sec. Referring the sun's motion to this origin we find its velocity to be 13.8 km./sec. towards the apex $A_0 = 267°$; $D_0 = +32°$.

3. If we exclude the very brightest F stars, many of which are Cepheid variables and stars of similar spectra, and take together stars of types A_7 to F_9 of absolute magnitude 0 to 6, we find that these stars divide themselves into two streams. One of these coincides with the general group of giant stars, as can be seen from the agreement of its condensation-points with that of the giant stars of other spectral types while the other stream moves nearly parallel to the Taurus Group. This group[6] is moving towards the point $\alpha = 92°$; $\delta = +7°$ with a velocity of 44 km./sec. or in galactic coördinates with our adopted origin $x = -26$, $y = +15$, $z = +4$ km./sec. This stream comprises the majority of the stars of F type and its existence seems to indicate that a large number of these stars in all regions of the sky are traveling with a velocity nearly identical with that of the Taurus Group. Even among the brightest F stars as well as the G-type giants the presence of members of this group is indicated by the protruding form of the equifrequential surfaces in this region. This stream might be identified with Kapteyn's First Drift although the galactic latitude is not zero.

4. The dwarf stars among which have been included those with absolute magnitudes fainter than 3.0 seem to form a group of their own as regards their motions. The distribution of the velocities is ellipsoidal, the major, intermediate and shortest axes having the galactic longitudes 162°, 70° and 291° and latitudes + 8°, + 9° and + 77°, respectively. The most frequent velocity is $x = -19$; $y = -2$; $z = +1$ km./sec. which nearly coincides with Kapteyn's First Drift, while his Second Drift has been swallowed up by the ellipsoidal stream of giant stars.

There is a very marked asymmetry in the distribution of the velocities around the most frequent velocity-vector. The stars of high velocity show a tendency to avoid the first quadrant of the xy plane. This asymmetry produces a systematic difference between the most frequent velocity and the mean of the velocity-components (the centroid). If we omit the stars of most rapid motion the centroid falls very near the origin and gives us nearly the standard solar velocity. That this asymmetry is not due to the selection of stars of large proper motion moving opposite to the sun is

shown by a study of the velocity-distribution of stars having very high speeds. In the diagram in figure 2 are plotted the projections in the xy plane of the velocities of stars having space motions larger than 100 km./sec. The effect of the selection of proper motions relative to the sun instead of to our adopted origin is then insignificant, and we find that whatever the spectral types or the absolute magnitude, if the velocity of the star

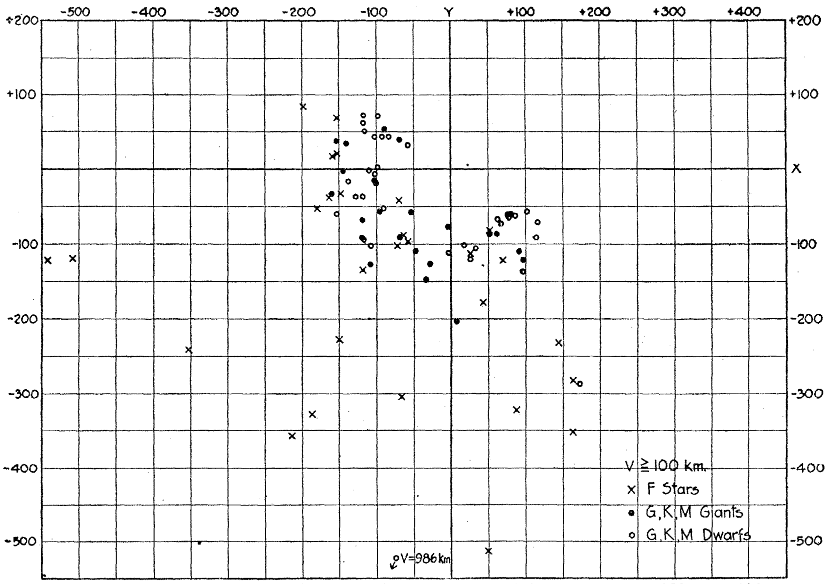

FIG. 2

Projections in the galactic plane of the velocities of stars having a speed greater than 100 km. per second.

is great, there is an avoidance of the first quadrant in the xy plane. No star of speed higher than 100 km./sec. is moving in this direction, but a large proportion of them are moving toward the third quadrant, a result in agreement with that found by Adams and Joy[3] for stars of high radial velocity.

[1] *Mt. Wilson Contr.* No. 199, *Astroph. J., Chicago,* **53,** 1921 (13).

[2] *London, Mon. Not. R. Astron. Soc.,* **79,** 1919 (201).

[3] *Mt. Wilson Contr.* No. 163, *Astroph. J.,* **49,** 1919 (179).

[4] Kapteyn, Address before St. Louis Exposition Congress, 1904.

[5] Schwarzschild, *Göttingen Nachrichten,* 1907, p. 614. A summary of the Theories is given in Eddington's, *Stellar Movements and the Structure of the Universe,* London, 1914.

[6] Hertzsprung, *Astron. Nachrichten,* **209,** 1918.